Test 1A - MTH-1310
Dr. Graham-Squire, Spring 2017

Name: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____
(signature)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.

2. Clearly indicate your answer by putting a box around it.

3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on the first 5 questions of the test, however you should still show all of your work. No calculators are allowed on the last 5 questions.

4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.

5. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

6. Make sure you sign the pledge.

7. Number of questions = 10. Total Points = 55.

1. (6 points) Weasel Enterprises produces contraptions made out of LEGOs. The cost function is given by
$$C(x) = x^3 - 15x^2 + 136.8$$
and the revenue function is given by
$$R(x) = x^3 - 15.01x^2 + 2.42x$$
where x is the number of contraptions produced, and both C and R represent <u>hundreds</u> of dollars (in either cost or revenue)

(a) Find the profit function $P(x)$.

(b) Find the marginal profit function. At what production level will marginal profit equal zero?

(c) Based on your answers from part (c), what is the optimal number of LEGO contraptions for Weasel Enterprises to produce? Explain your answer by interpreting the meaning of your answer from (b).

2. (6 points) Clifford is on the moon, and he throws a ball into the air. The height in feet of the ball above ground is given by

$$h(t) = -2.65t^2 + 10t + 4$$

where $t = 0$ is the point at which Clifford releases the ball. <u>Use calculus</u> to find each of the following, and show your work to receive full credit!

(a) Find the acceleration of the ball.

(b) At what time does the ball hit the ground? Round to the nearest 0.01 seconds.

(c) What is the highest point the ball reaches in the sky? Round to the nearest 0.01 feet.

3. (6 points) A bird is on an island 100 miles from a straight coastline, and the town of P is the closest point on the coastline. The bird's nesting area is 1000 miles down the coast from P. The bird is trying to figure out the fastest way to fly from the island to her nest, and she is aware that she flies at a speed of 10 miles/hour when flying over water, and 12 miles/hour when flying over land. Round answers to nearest 0.1 hours.
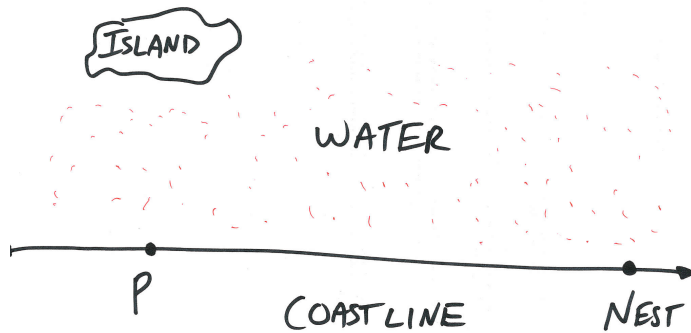
(a) If she flies the 100 miles directly from the island to P, then straight 1000 miles along the coastline to her nest, how long will it take her (in hours)?

(b) If she flies diagonally from the island directly to the nest (all over water), how long will it take her?

(c) If she flies diagonally from the island to a spot halfway between her nest and P (all over water), then 500 miles on land to her nest, how long will it take her?

(d) Suppose the bird lands at a point x miles along the coast from P. Write a function to calculate how long it will take the bird to fly from the island to that point (over water) and then the rest of the way along land.

4. (6 points) Consider the graph of $f(x)$ below, then answer the questions. If something does not exist, you can answer DNE.
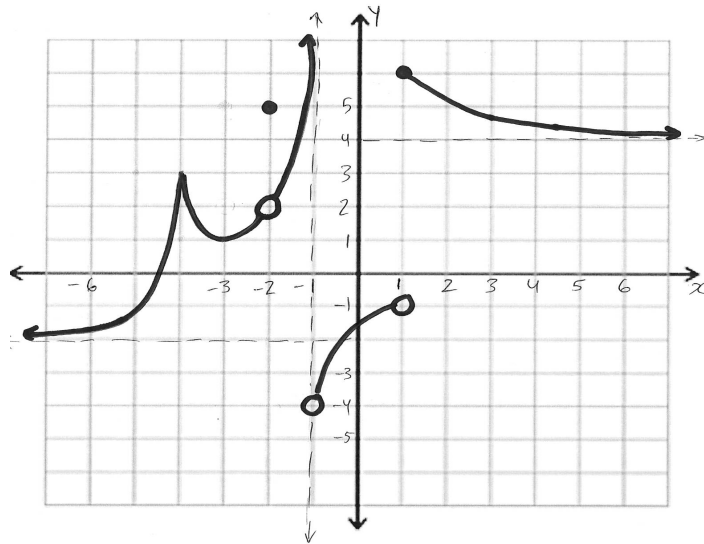
Calculate:

(a) $\displaystyle\lim_{x\to(-4)^+} f(x)$

(b) $\displaystyle\lim_{x\to(-2)} f(x)$

(c) $\displaystyle\lim_{x\to(-1)^-} f(x)$

(d) $\displaystyle\lim_{x\to1^-} f(x)$

(e) $\displaystyle\lim_{x\to\infty} f(x)$

(f) An x-value where $f'(x)$ is undefined, and WHY it is undefined there.

5. (3 points) Calculate the limit. It is fine to use a calculator to check your work, but you should show your work and/or explain your reasoning as if you did not have a calculator. Make sure to use correct notation! If something does not exist, you can answer DNE.

$$\lim_{x \to (-\infty)} \frac{3 - 2x^2 + 5x^3}{x^4 - 3x}$$

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6. (6 points) Graph the following piecewise function:

$$f(x) = \begin{cases} 2x + 4 & \text{if } x < -1 \\ -x - 2 & \text{if } -1 \leq x \leq 2 \\ x^2 - 4x & \text{if } x > 2 \end{cases}$$
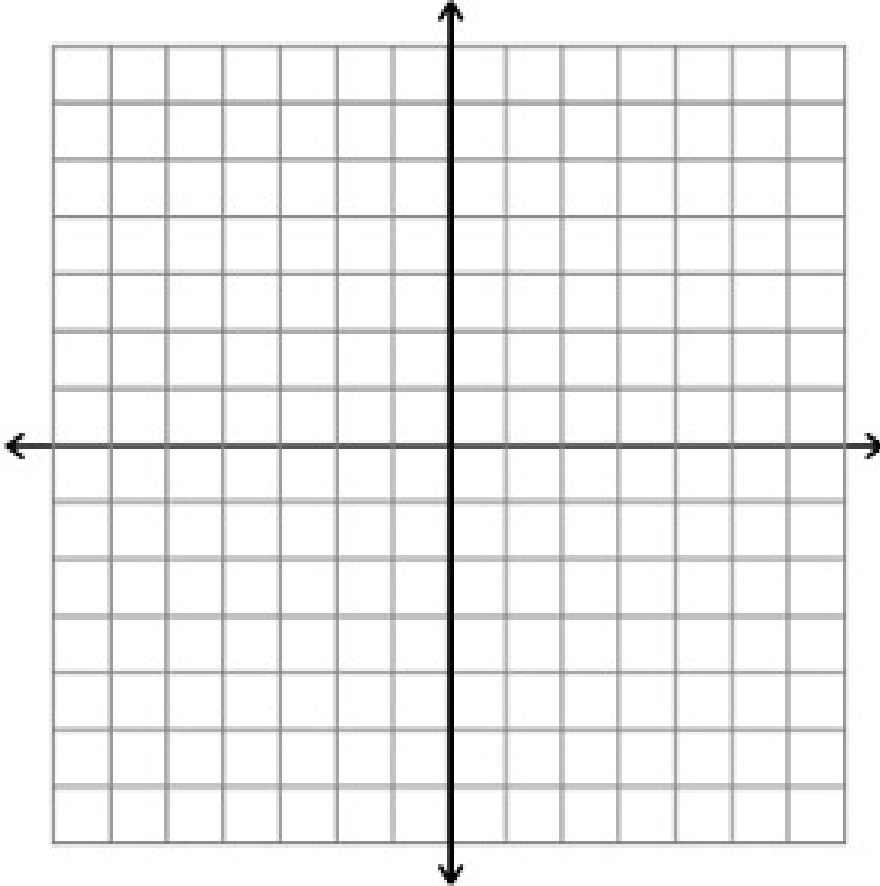
7. (5 points) Take the derivative and simplify your answer by multiplying out expressions and combining like terms (if necessary). Your simplified answer should end up in the form of a polynomial.

$$f(x) = x^2(x+7)(x^2 - 3x + 5)$$

8. (6 points) Calculate the derivative (in whatever manner you feel is most appropriate) and simplify your answer as much as possible.

$$\frac{x^8 - 3x^6 + 4 + \frac{1}{x^2}}{x^3}$$

9. (6 points) Calculate the limits. Make sure to show your work and/or explain your reasoning, and use correct notation! If something does not exist, you can answer DNE.

(a) $\lim\limits_{x \to 4} \dfrac{x^2 + 3x - 28}{x^2 - 8x + 16}$

(b) $\lim\limits_{h \to 0} \dfrac{3(x + h)^2 - 7(x + h) - (3x^2 - 7x)}{h}$

10. (5 points) Calculate the derivative of the function below. You do NOT need to simplify your answer.
$$\left(3 + (x^2 - 7)(x^3 + 4x)^4\right)^{17}$$

Extra Credit(1 point) Calculate the derivative of the function $f(x) = (42(15 + 3))^7$, and explain your answer.